Assessing GDP and Inflation Probability Forecasts Derived from the Bank of England Fan Charts

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Abstract

Density forecasts, including the pioneering Bank of England 'fan charts', are often used to produce forecast probabilities of a particular event. We use the Bank of England's forecast densities to calculate the forecast probability that annual rates of inflation and output growth exceed given thresholds. We subject these implicit probability forecasts to a number of graphical and numerical diagnostic checks. We measure both their calibration and their resolution, providing both statistical and graphical interpretations of the results. The results reinforce earlier evidence on limitations of these forecasts and provide new evidence on their information content and on the relative performance of inflation and GDP growth forecasts. In particular, GDP forecasts show little or no ability to predict periods of low growth beyond the current quarter, due in part to the important role of data revisions.

Key words: calibration, density forecast, probability forecast, resolution, sharpness

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1. Introduction

Forecasts of the probability that a specific event will occur have attracted increasing recent interest; see Gneiting (2008) for a discussion and overview. While many series of such probability forecasts have been subject to careful evaluation (particularly in meteorological contexts), important economic series have received less precise attention. The present paper addresses one well known source of economic probability forecasts, arising from the Bank of England's fan charts, and subjects them to a number of diagnostic checks. We examine both inflation and gross domestic product (GDP) growth forecasts; the latter are particularly important, because much economic behaviour (such as investment, hiring, major purchasing decisions) is influenced by expectations of near-term growth. One noteworthy finding is that the GDP growth forecasts have little ability, beyond that of the unconditional distribution of GDP growth, to distinguish high- and low- probability growth outcomes.

Since their introduction in the 1993 Inflation Report, the Bank of England's probability density forecasts ("fan charts") for inflation, and later output growth, have been studied by a number of authors. Wallis (2003, 2004) and Clements (2004) studied the inflation forecasts and concluded that while the current and next-quarter forecast seemed to fit well, the year-ahead forecasts significantly overestimated the probability of high inflation rates. Elder, Kapetanios, Taylor and Yates (2005) found similar results for the inflation forecasts, but also found significant evidence that the GDP growth forecasts do not accurately capture the true distribution of risks to output growth at very short horizons. Noting the hazards of drawing firm conclusions from small samples, these authors suggested that "the fan charts gave a reasonably good guide to the probabilities and risks facing the MPC [monetary policy committee]." They also explored the role of GDP revisions in accounting for GDP forecast errors and noted that the dispersion associated with predicted GDP outcomes was increased as a result of their research. Dowd (2007) concluded that uncertainty in inflation was substantially overestimated, and Gneiting and Ranjan (2011) reached a similar conclusion with respect to longer-horizon forecasts in particular. Dowd (2008) examined the GDP fan charts and found that while short-horizon forecasts appear to capture the risks to output growth poorly, results for longer horizon forecasts are sensitive to the vintage of data used to evaluate the forecasts, a point to which we will return below.

Throughout this evaluative work, the focus has been on whether the risks implied by the Bank's fan charts are well matched (in a statistical sense) by the frequency of the various inflation and output growth outcomes. Gneiting, Balabdaoui and Raftery (2007) refer to this property as 'probabilistic calibration'. As we will see below, it is well known that different density functions may satisfy this correct calibration property yet convey quite different amounts of information; see Corradi and Swanson (2006) and Mitchell and Wallis (2011) for a discussion. Mitchell and Wallis note that this extra information has been referred to as 'sharpness', 'refinement' or 'resolution' in various contexts and note its relationship to the Kullback-Leibler Information Criterion. Although forecast sharpness or resolution is desirable, no empirical studies of the Bank's density forecasts have investigated this property.

Instead of the full forecast density, we work with the implied probabilistic forecasts:

we compute the forecast probabilities of failing to achieve the Bank's inflation target, or of GDP growth falling below a fixed threshold. (The methods that we use to do so can of course be applied to probability forecasts for other thresholds simply by integrating under different regions of the density forecast; different choices of threshold allow one to focus on different parts of the forecast distribution.) This is similar in spirit to Clements' (2004) examination of the fan chart's implied interval forecasts. The Bank of England also examines such interval forecasts from time to time; see for example Table 1 (p. 47) of the August 2008 Inflation Report. We investigate both the calibration of the probabilistic forecasts (the degree to which predicted probabilities correspond with true probabilities of the outcomes), and their resolution (their ability to discriminate among different outcomes).

In contrast with earlier evaluations, our results provide strong evidence of a mis-calibration of the inflation forecasts at very short horizons, even though the degree of mis-calibration appears to be small. Despite the much shorter sample available for the GDP forecasts, we again find significant evidence of mis-calibration and its magnitude appears to be much larger than for inflation. Results on the discriminatory power of the forecasts shows that inflation forecasts appear to have important power to distinguish high- and low- probability cases up to horizons of about one year. Perhaps most importantly we find that the resolution of the GDP forecasts, upon which many important decisions are based, is almost negligible beyond a one-quarter horizon.

2. Data and forecasts

The Bank of England's Inflation Report provides density forecasts of inflation and, more recently, output growth in the form of 'fan charts'. Fan charts for RPIX (retail prices index excluding mortgage payments) inflation were published from 1993Q1 to 2004Q1, when they were replaced by CPI (consumer price index) inflation fan charts. Both measure inflation as the percentage change in the corresponding price index over four quarters. See Wallis (1999) for a careful discussion of the interpretation of these charts; note in particular that the different bands do not correspond straightforwardly with quantiles in the general, asymmetric, case. The GDP fan chart was first published in the 1997Q3 report and also forecasts the total percentage growth over 4 quarters. In addition to providing forecast distributions for roughly 0 to 8 quarters into the future, from the 1998Q1 Inflation Report onwards forecasts are provided conditional on the assumption of either fixed interest rates or a "market-expectation-based" interest rate profile; the two assumptions typically provide similar results, and below we will present results for the market interest rate case only (results for fixed interest rate forecasts are qualitatively very similar). Construction of probability forecasts from the Bank's fan charts is described in the Appendix.

For both inflation and GDP growth, we use all available forecasts up to and including that published in 2010Q1. We measure inflation and output growth outcomes using the 2010Q1 vintage data series, and for GDP we also use the unrevised first estimates of annual GDP growth.

Taking account of data revision can change materially the apparent performance of GDP growth forecasts. While inflation (or price level) data are not revised to any substantial extent, revisions in GDP growth are often comparable in magnitude to GDP growth itself.

If our aim in forecasting GDP growth is to predict the actual change in national output (as opposed to predicting the next initial estimate) then we wish to evaluate performance by the best estimate of that actual change, i.e. by the latest, last-revised number. Forecasters do not have the benefit of this latest revision, of course. Working as they do with preliminary data, they may sometimes produce forecasts which have some predictive power for the next preliminary data points, but very little for the final estimates of those data points. Below we present evidence of this phenomenon in the Bank of England (BoE) forecasts.

We report results for nine horizons, zero (the 'nowcast' for the current quarter) through eight.

Figure 1 is intended to convey some descriptive information about the probability forecast series with which we work. The left-hand and middle panels of the figure (the right-hand panels are described below) show the implications of the BoE's density forecasts for the probabilities that annual real GDP growth is less than the 2.5% threshold mentioned above, and that annual inflation (based on RPIX or CPI) is less than the Bank's target value. Note that the Bank has a scalar target value as well as a band within which inflation outcomes are considered acceptable; we present results based on each of these indicators below.

Each point in these panels corresponds with the implied forecast probability (on the vertical axis) that inflation or output growth will be less than the chosen threshold, at the forecast horizon given on the horizontal axis. The '+' symbols represent cases where the outcome for which the probability is forecast occurred (e.g. forecast is of probability of falling below a threshold, and the eventual outcome was below the threshold), while the circles are cases in which the outcome did not occur. One clearly observable feature is that the dispersion of these forecasts declines with increasing horizon, as is appropriate; the declining value of conditioning information with lengthening horizon makes it more difficult to distinguish high-probability and low-probability states.

Ideal forecasts would have assigned probability one to all the '+'s and probability zero to the circles. Instead, for GDP growth we observe several high probability circles and low probability '+'s (see horizons 2-4 in particular). We also find most outcomes clustered in the center of the probability range at horizons 4-8. The probabilistic outcomes for inflation show similar features with respect to changes across horizons, but at the shorter horizons we see a more marked concentration of '+'s at the higher probabilities and of circles at the lower, suggesting that the inflation forecasts had more discriminatory power than the GDP forecasts, at least at the short horizons. Below we will present direct evidence on this point.

In the next section, we review some of the literature on density forecast evaluation before focusing on tests of probabilistic forecasts and properties of forecast calibration and resolution or sharpness.

3. Probability forecast evaluation

3.1 Predictive densities

Let X be a random variable with realizations x_t and with probability density and cumulative distribution functions $f_X(x)$ and $F_X(x)$ respectively. Then for a given sample $\{x_t\}_{t=1}^T$, the corresponding sample of values of the CDF, $\{F_X(x_t)\}_{t=1}^T$, is a U(0,1) sequence. This well-known result (often termed the probability integral transform of $\{x_t\}_{t=1}^T$) is the basis of much predictive density testing, following pioneering work by Diebold, Gunther and Tay (1998). These authors noted that if the predictive density $\hat{f}_X(x)$ is equal to the true density, then using the predictive density for the probability integral transform should produce the same result, i.e. a U(0,1) sequence. This allows us to test whether a given sequence of forecast densities could be equal to the true sequence by checking whether $\{\hat{F}_X(x_t)\}_{t=1}^T$ (i.e. the sequence of CDFs of the realized values using the forecast densities) is U(0,1).

The right-hand panels of Figure 1 show histograms of the probability integral transforms (PIT's) for the Bank's GDP growth and inflation forecasts. As these are U(0,1) under the null of correct specification of the conditional density, the histograms should show roughly the same proportion of observed forecasts in each of the ten cells. In order to represent compactly the results for nine forecast horizons (0–8 inclusive) in each part of the figure, we have indicated the outcomes with a colour coding; each row of the figure represents a different horizon, and each column a particular bin with width 0.1. Uniformly distributed results would imply a frequency of 0.1 in each bin, and therefore a uniform colour in the figure. Values well below 0.1 show up as dark blue, and well above 0.1 as red.

While some sampling variation is inevitable, the results are far from uniformity. GDP growth forecasts often show an excessive number of values in the highest cell (near 1) at short horizons, an insufficient number at long horizons, and an insufficient number of values near zero at virtually all horizons. Similar patterns are observable in inflation forecasts, albeit to a lesser degree.

Note that values of the probability integral transform that are too low (shades of blue in the right-hand panels of Figure 1) near the extremes are an indication of forecast densities that are too dispersed: in such cases, actual outcomes do not reach the tails of the forecast density as often as they would relative to the true conditional density, and so observed outcomes tend to fall in intermediate regions of the forecast density.

If the sequences represented in the rows are assumed to be independent, the U(0,1) condition is easily tested with standard tests (such as a Kolmogorov-Smirnov one-sample test.) The independence is unrealistic in many economic applications, however. In particular, violation is almost certain for multiple-horizon forecasts as the $h-1$ period overlap in horizon-h forecasts induces an MA($h-1$) process in the forecast errors; see for example Hansen and Hodrick (1980). The inferential problem is therefore more difficult: test statistic distributions are affected by the form of dependence.

3.2 Probabilistic forecasts

Rather than analyse the entire predictive density, here we examine probabilistic forecasts implied by the BoE forecasts; e.g., the probability that an outcome (inflation or output growth) will be below some threshold. This implies a loss of information relative to the full density forecast. Of course, if the forecasts are of events of particular economic interest, this loss of efficiency may be inconsequential. Probabilistic forecasts also permit a particularly simple decomposition that is useful for interpreting forecast behaviour and the sources of forecast errors.

Following the notation of Murphy and Winkler (1987), let x be a 0/1 binary variable representing an outcome and let $\hat{p} \in [0, 1]$ be a probability forecast of that outcome. Forecasts and outcomes may both be seen as random variables, and therefore as having a joint distribution; see e.g. Murphy (1973), from which much subsequent work follows.

Numerous summary measures of probabilistic forecast performance have been suggested, including loss functions such as the Brier score (Brier, 1950, Murphy 1973) which is a mean squared error (MSE) criterion. Since the variance of the binary outcomes is fixed, it is useful to condition on the forecasts: in this case we can express the mean squared error $E((\hat{p} - x)^2)$ of the probabilistic forecast as follows:

$$E(\hat{p} - x)^2 = E(x - E(x))^2 + E(\hat{p} - E(x|\hat{p}))^2 - E(E(x|\hat{p}) - E(x))^2. \qquad (1)$$

Note that the first right-hand side term, the variance of the binary sequence of outcomes, is a fixed feature of the problem and does not depend on the forecasts. Hence all information in the MSE that depends on the forecasts is contained in the second and third terms on the right-hand side of (1). The MSE is of course only one of many possible loss functions, and is inappropriate in some circumstances. We focus on it here because there is no consensus on the precise form of an appropriate loss function for an inflation-targetting central bank and because we argue that the decomposition it presents is helpful in understanding forecast performance.

3.3 Calibration and resolution

We will call the first of the terms involving $\hat{p}$ in (1),

$$E(\hat{p} - E(x|\hat{p}))^2, \qquad (2)$$

the (mean squared) *calibration error:* it measures the squared deviation of the predicted probability from the true conditional probability of the event. (This quantity is often called simply the 'calibration' or 'reliability' of the forecasts. We prefer the term calibration *error* to emphasize that this quantity measures deviations from the ideal forecast, and we will use 'calibration' to refer to the general property of conformity between predicted and true conditional probabilities.) If for any forecast value $\hat{p}_i$ the true probability that the event will occur is also $\hat{p}_i$, then the forecasts are perfectly calibrated. If for example we forecast that the probability of GDP growth below some level g in the next quarter is 50%, and if over all occasions on which we would make this forecast the proportion in which growth is below g is indeed 50%, and if this match holds for all other forecast values (of GDP growth below other levels h), then the forecasts are perfectly calibrated. Note that perfect calibration can be achieved here by setting $\hat{p} = E(x) = 0.5$, the unconditional probability, since the calibration is evaluated at the possible values or over the range of values that the probability forecast takes on.

Calibration has typically been investigated using histogram-type estimates of the conditional expectation, grouping probabilities into cells. Instead, we use the approach suggested by Galbraith and van Norden (2011), who show how to use smooth conditional expectation functions estimated via kernel methods to estimate $E(x|\hat{p})$ and test for mis-calibration.

The last term on the right-hand side of (1), $E(E(x|\hat{p}) - E(x))^2$, is called the forecast *resolution*, and measures the ability of forecasts to distinguish among relatively high-probability and relatively low-probability cases. High resolution implies that the conditional expectation of the outcome often differs substantially from its unconditional mean: the forecasts successfully identify cases in which probability of the event is unusually high or low. The resolution enters negatively into the MSE decomposition: high resolution lowers MSE. To return to the previous example, the simple forecast that always predicts a 50% probability of growth below g, where 50% is the unconditional probability, will be correctly calibrated but have zero resolution. Consider also two forecasters A and B, who issue forecasts of GDP growth below g of 0.4, 0.5 or 0.6 each with probability $\frac{1}{3}$ (A) and of 0.1, 0.5 or 0.9 each with probability $\frac{1}{3}$ (B). Assume that each of the sets of forecasts A and B are correctly calibrated. B's forecasts are the more useful, suggesting as they do quite high or quite low probability of growth below g two-thirds of the time, whereas A only weakly distinguishes cases where growth is likely or unlikely to be below normal. A's resolution is $\frac{1}{3}(0.4 - 0.5)^2 + \frac{1}{3}(0.6 - 0.5)^2 = \frac{2}{3}(0.01)$; B's resolution is $\frac{1}{3}(0.1 - 0.5)^2 + \frac{1}{3}(0.9 - 0.5)^2 = \frac{2}{3}(0.16)$. Perfect forecasts would have resolution equal to variance.

The calibration error has a minimum value of zero; its maximum value is 1, where forecasts and conditional expectations are perfectly opposed. The resolution also has a minimum value of zero, but its maximum value is equal to the variance of the binary outcome process. In order to report a more readily interpretable measure, scaled into $[0, 1]$, we divide the resolution by this maximum possible value. The variance of a 0/1 random variable with mean (proportion of 1's) μ is $\mu(1-\mu)^2 + (1-\mu)\mu^2$; therefore we report the scaled value

$$\frac{E(E(x|\hat{p}) - \mu)^2}{var(x)} = \frac{E(E(x|\hat{p}) - \mu)^2}{\mu(1-\mu)^2 + (1-\mu)\mu^2} \in [0, 1]. \tag{3}$$

The information in the resolution is correlated with that in the calibration; the decomposition just given is not an orthogonal one (see for example Yates and Curley 1985). However the resolution also has useful interpretive value which we will see below in considering the empirical results. The calibration and/or resolution of probabilistic economic forecasts have been investigated by a number of authors, including Diebold and Rudebusch (1989), Galbraith and van Norden (2011), and Lahiri and Wang (2007). The meteorological and statistical literatures contain many more examples; some recent contributions include Hamill et al. (2003), Gneiting et al. (2007), Ranjan and Gneiting (2010) and Thorarinsdottir and Gneiting (2010). We now use these methods to examine the BoE forecasts.

4. Empirical results

Table 1 and Figures 2 and 3 contain sample counterparts of the theoretical quantities described earlier. We begin by interpreting the graphical diagnostics in Figures 2 and 3.

Figure 2 provides another way of understanding the resolution of the probability forecasts that does not require estimation of the conditional expectation. For horizons of zero, two and four quarters each of the panels presents a pair of empirical CDF's (broken and solid lines, in the same colour) of the forecast probabilities that an event will occur: growth below threshold, or inflation within bands. The solid-line CDF's apply to cases for which the event did, in the end, occur (growth below threshold or inflation within given bands), and the broken-line CDF's apply to cases for which the event did not occur. In a near-ideal world, these forecast probabilities should be near one in cases where the predicted event did occur, and near zero when it did not; correspondingly, the broken-line CDF's should lie close to the upper horizontal axis (indicating that the distribution contains mainly values near zero), and the solid-line CDF's would lie close to the lower horizontal axis (indicating that the distribution contains mainly values near one). More generally, good probability forecasts will discriminate effectively between the two possible outcomes, and the two empirical CDF's of the same colour should be widely separated in each panel. At longer horizons, the value of conditioning information declines and this separation becomes more difficult to achieve; we therefore expect to see the pairs of CDF's less widely separated for longer horizons.

This pattern of reduced separation with horizon is in fact readily observable; at the four-quarter horizons, we observe little separation of the CDF's for either forecast series. However, at shorter horizons, we observe clear distinctions between cases. Inflation forecasts show greater separation at low horizons, suggesting greater resolution; note however that there are few cases in which the inflation outcome failed to lie within bands, so the empirical CDF's for the cases of outcome out of the band show very broad steps. GDP forecasts show substantial separation when evaluated against the preliminary data, but evaluated against the latest estimates of GDP growth at the relevant period, there is little separation even at short horizons. The GDP growth results mirror the low 'content horizon' on U.S. and Canadian GDP growth point forecasts reported by, for example, Galbraith (2003) and Galbraith and Tkacz (2007): that is, forecasts of GDP growth generally do not improve markedly on the simple unconditional mean beyond about one or two quarters into the future. Galbraith and van Norden (2011) estimate conditional expectation functions for the Survey of Professional Forecasters probabilistic forecasts for US real output contractions using methods very similar to those used here, and find that forecasts appear to be essentially equivalent to unconditional forecasts at horizons of more than two quarters, which implies zero forecast resolution.

Figure 3 gives a different perspective on calibration error and resolution by showing their importance relative to mean squared forecast error (MSFE). Recalling that (1) decomposes MSFE into mean-squared calibration error, resolution and the variance of outcomes, we can divide by the unconditional variance of the outcomes, $V_x = E(x - E(x))^2$, and re-arrange to obtain

$$\frac{E(\hat{p}-x)^2}{V_x} - \frac{E(\hat{p}-E(x|\hat{p}))^2}{V_x} + \frac{E(E(x|\hat{p})-E(x))^2}{V_x} = 1. \qquad (4)$$

The first term is the scaled MSFE, the second is the scaled calibration error and the third is the scaled resolution. Figure 3 shows how each of these three terms vary across forecast horizons. Since calibration error enters negatively, the figure shows negative scaled calibration error; the three components then always sum to one.

Using preliminary data for GDP outcomes, we see that while calibration error is roughly constant across horizons, resolution decreases steadily as the horizon increases, causing a roughly equivalent increase in MSFE. Resolution at horizons beyond 4Q is close to zero. However, using revised data for GDP outcomes, we see much larger calibration error at horizons up to one year and resolution that is close to zero at all horizons. The result is an MSFE that *exceeds* the variance of GDP outcomes at all horizons. (This is not uncommon with GDP growth forecasts; see for example Galbraith and Tkacz 2007.) This implies that while the Bank forecasts appear to have some information content at shorter horizons, this content is an artefact of using unrevised data. Using revised data (so that we consider forecasting the final estimates of true outcomes), the forecasts contain little information: this may well be a result of the low information content of the initial-release GDP information available to economic forecasters.

For inflation, shown in the lower panels, the results are broadly similar regardless of whether we examine the probability that inflation will be within the target band or that it will exceed the target level. In both cases, there is substantial resolution at the very shortest horizons which declines to roughly zero for forecast horizons of four quarters or more. (Recall that the measure of inflation being forecast is the four-quarter change in the price level. At forecast horizons of less than four quarters, therefore, some fraction of this four-quarter change has already been observed.) While there is some variation in calibration error, MSFE is dominated by the drop in forecast resolution.

To judge whether any of the deviations from perfect calibration or from zero resolution are statistically significant, we require formal tests of the null hypotheses, respectively $E(x|\hat{p}) = \hat{p}$ (correct calibration) and $E(x|\hat{p}) = E(x)$ (zero resolution). The results of these tests are given in Table 1, which contains the decomposition (1) of the variance of outcomes x into MSFE, squared calibration error and resolution, as well as the results of formal tests of the null hypotheses that (a) calibration error is zero, and (b) resolution is zero.

As the decomposition of variance has been described above, consider now the tests of hypotheses (a) and (b). The tests are based on the facts that $E(x|\hat{p}) = \hat{p}$ implies correct calibration, and that $E(x|\hat{p}) = a$, a constant, implies zero resolution. To test calibration, again as in Galbraith and van Norden (2011), we estimate the model $x_i = a + b\hat{p}_i + c\hat{p}_i^2 + \epsilon_i$, and jointly test $H_0 : a = 0, b = 1, c = 0$ with a χ^2_3- distributed Wald statistic, using robust (Newey-West) standard errors in the computation. The test of zero resolution is of $H_0 : b = 0 \text{ and } c = 0$, and is also computed as a (χ^2_2) Wald statistic using robust standard errors. The $p-$values from these Wald tests are reported in Table 1.

The results for the GDP forecasts reinforce the graphical analysis. Results using preliminary GDP data are benign, with little or no significant evidence of mis-calibration at any forecast horizon, and with statistically significant forecast resolution at all horizons up to and including four quarters. Results using revised GDP data are much less encouraging; there is strongly significant evidence of mis-calibration at both longer and shorter forecast horizons, and there is no significant evidence of positive forecast resolution for anything beyond the current quarter. These observations are consistent with the conditional empirical CDFs shown in Figure 2 and the decompositions in Figure 3. A natural interpretation is that Bank of England forecasts of final revision data are mis-calibrated at least in part because of systematic features of the data revision process. Elder et al. (2005, Appendix C) note that GDP revisions appeared to explain the bias and much of the forecast error in the Bank's short-term forecasts. Starting with the November 2007 Inflation Report, the Bank of England takes account of data revision in its GDP density forecasts and publishes density forecasts for data revisions. Because of the small number of sample points, we do not attempt to assess the impact of this change on the performance of the forecasts.

For the inflation level forecasts, we see that while there is strongly significant evidence of calibration errors at the longest and shortest horizons, there is no evidence of such errors for one to four quarter horizons. The resolution tests confirm the analysis presented in Figures 2 and 3; while there is significant resolution at horizons up to three quarters, there is no detectable resolution beyond that point. Results for the inflation interval forecasts are quite similar with respect to forecast resolution. However, there is strong and widespread evidence of forecast mis-calibration.

To summarize these empirical results: for inflation forecasts, deviations from correct calibration appear to be small, although nonetheless statistically significant at a number of forecast horizons. GDP growth forecasts, particularly of revised outcomes, produce much larger estimated deviations from correct calibration. However, only a subset of the observed deviations are statistically significant, perhaps because of the limited sample size.

Resolution falls rapidly with forecast horizon, is higher for inflation forecasts, and for GDP is in most cases difficult to distinguish statistically from zero.

These results, particularly at longer horizons, reflect differences in inflation and GDP growth forecasts observed in other contexts: the usefulness of conditioning information allowing us to make forecasts appears to decay much more quickly for GDP growth, and the persistence in the data is much lower.

5. Discussion

By focusing our attention on the probabilistic forecasts implied by the Bank of England's fan charts, we have evaluated their performance on a number of criteria relevant to policy-makers: do they correctly capture the risk of high or low growth? the risk of high or low inflation? the risk that inflation will be outside the Bank's target band? Concentrating on these probabilistic forecasts uses a subset of the information contained in the complete density, but allows us to examine the extent to which these forecasts contain useful information about particular types of outcome.

The resolution of GDP forecasts, in particular, has important public policy implications.

Forecasts of the probability that the economy will enter, remain in, or exit a period of slow growth or recession may have substantial effects on individual and firm behaviour. However, these forecasts do not appear to be very informative. In the case of the Bank of England forecasts examined here, the fact that there is some evidence of near-term resolution when results are evaluated using preliminary data, but almost none using latest-vintage data, suggests that the problem lies not with forecast methods, but with the high noise content of initial-release data. This evidence, consistent with that of Elder et al. (2005), underscores the importance that data revision can play in the design and interpretation of economic forecasts.

The fact that inflation probability forecasts show more resolution, and that initial price level data are *not* substantially revised, is also consistent with this observation. However, the evidence of resolution in the inflation forecasts may be attributable to the fact that the Bank is forecasting the four-quarter change in prices; there is no significant evidence of forecast resolution for inflation at longer horizons.

As in earlier studies, we also find some statistically significant evidence of differences between the predicted probabilities and the actual probabilities of the subsequent outcomes, and we are able to relate the degree of this calibration error to overall forecast performance. For most of the cases that we examine, calibration error plays only a minor role in explaining changes in MSFE across different forecast horizons; the dominant factor is the fall in forecast resolution with increasing horizon. The only exception to this result lies in the evaluation of GDP forecasts with revised data, where again we find low forecast resolution at *all* forecast horizons.

A lack of resolution is important for forecast users to understand. For example, in the past the Bank of England has reacted to its own analysis of forecast performance by adjusting the dispersion used in its density forecasts; for example, see Elder et al. (2005) or the Bank's August 2009 Inflation Report. This does not address the problem of the lack of forecast resolution. Whether sufficient information exists to produce forecast densities for GDP growth which have useful resolution at the longer horizons used by the Bank of England is an open question: low or zero resolution in GDP growth forecasts appears to be common. This lack of resolution does not imply that Bank of England forecasts use information less efficiently than competitors, but does imply that these forecasts contain little information beyond the unconditional distribution of GDP growth. Little reliance, therefore, should be placed on such forecasts even at quite short horizons.

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Appendix: Construction of probability forecasts

While the Bank of England's fan charts provide only a visual guide to the degree of uncertainty that the Bank associates with its forecasts, they are based on an explicit parametric model. Future outcomes are assumed to follow a 'two-piece normal' or 'bi-normal' distribution, the behaviour of which is fully characterized by three parameters: a mean μ, a measure of dispersion σ, and a parameter which controls skewness, γ.These parameters therefore allow us to estimate the implied forecast probabilities that inflation or GDP growth would exceed any given threshold or fall within any given range. Spreadsheets containing the parameter settings for all of the published fan charts are publicly available on the BoE's web site (presently at http://www.bankofengland.co.uk/publications/inflationreport/irprobab.htm).

The Bank of England publishes five values (mode, median, mean, uncertainty and skew) for each of its density forecasts. The mode and uncertainty correspond with μ and σ, but γ is only indirectly related to the the published skew, S, via

$$\gamma = sgn(S)\left[1 - 4\left(\frac{\sqrt{1+\pi S^2}-1}{\pi S^2}\right)^2\right]^{\frac{1}{2}}, \tag{5}$$

with $sgn(S)$ denoting the sign of the skewness S.

The bi-normal distribution has the convenient property that its cumulative distribution function (CDF) can be rewritten as a function of the standard normal CDF; see Brittan, Fisher and Whitley (1998) and Wallis (2004, particularly Box A on p. 66) for a description of the distribution and its alternative parameterizations. Calculation of any probabilistic forecast implied by the Bank's density forecasts simply requires the standard normal CDF, the parameters $\{\mu, \sigma, \gamma\}$, and the points of evaluation ℓ_1 and ℓ_2 defining a probability $P(\ell_1 < x < \ell_2)$.

Notes to Table and Figures

Table 1

Table 1 reports sample estimates of quantities introduced in the text, pertaining to forecasts for the indicator variable which equals one when either average CPI inflation or GDP growth exceeds the indicated annual threshold rate over the indicated forecast horizon, and zero otherwise. MSFE denotes the mean squared forecast error, and 'MS calibration' is the sample estimate of the mean squared calibration error, (2). The scaled resolution is the estimate of the quantity given by equation (3).

Figure 1

Left and middle panels show probability forecasts; '+' represents cases of eventual outcome below threshold or within bands, circles are cases of outcome above threshold or outside bands. In right-hand panels each column represents one bin, 0–0.1, 0.1–0.2, ... 0.9–1.0, and each row one horizon, 0-8. Each square represents, via colour, the height of a histogram corresponding with the horizon and bin. Ideally, the probability integral transforms would yield a U(0,1) sequence, so that each row would show uniform values equal to 0.1.

Figure 2

In each panel, lines of the same colour denote the same horizon; for each horizon, the solid line represents the empirical CDF of probability forecasts for cases in which the event of interest did occur (e.g., inflation within bands) and the broken line represents cases in which the event of interest did not occur.

Figure 3

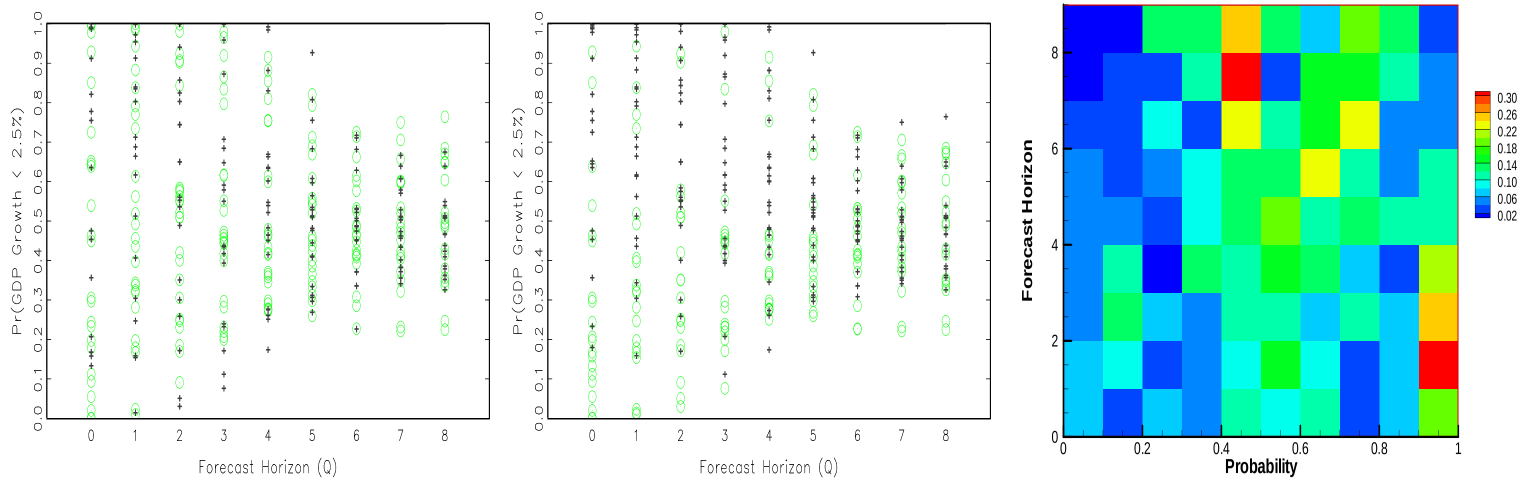
Each panel plots scaled values of mean squared forecast error, -1$\times$ calibration error, and resolution.

Table 1: Calibration and resolution of forecasts

	GDP growth: preliminary release data								
Horizon:	0	1	2	3	4	5	6	7	8
Outcome Variance:	0.245	0.243	0.241	0.238	0.240	0.242	0.243	0.245	0.243
MSFE:	0.122	0.158	0.172	0.195	0.217	0.240	0.243	0.263	0.266
Resolution:	0.122	0.084	0.065	0.061	0.049	0.020	0.006	0.007	0.014
MS calibration:	0.002	0.006	0.008	0.019	0.027	0.027	0.015	0.034	0.052
Scaled resolution $\in [0,1]$:	0.500	0.347	0.268	0.255	0.205	0.081	0.026	0.028	0.056
Calibration p-value	0.747	0.755	0.679	0.561	0.535	0.434	0.554	0.090	0.103
Resolution p-value	0.000	0.000	0.000	0.001	0.033	0.095	0.331	0.904	0.940
	GDP growth: 2010Q1 vintage data								
Horizon:	0	1	2	3	4	5	6	7	8
Outcome Variance:	0.247	0.248	0.249	0.250	0.250	0.250	0.250	0.249	0.249
MSFE:	0.275	0.292	0.298	0.310	0.281	0.270	0.262	0.264	0.278
Resolution:	0.031	0.021	0.018	0.011	0.016	0.003	0.001	0.006	0.011
MS calibration:	0.065	0.067	0.072	0.070	0.048	0.025	0.015	0.024	0.044
Scaled resolution $\in [0,1]$:	0.124	0.084	0.072	0.045	0.062	0.011	0.004	0.026	0.045
Calibration p-value	0.005	0.006	0.030	0.074	0.342	0.478	0.487	0.001	0.000
Resolution p-value	0.005	0.256	0.617	0.871	0.775	0.880	0.893	0.643	0.169
	Inflation: target threshold								
Horizon:	0	1	2	3	4	5	6	7	8
Outcome Variance:	0.250	0.250	0.250	0.250	0.249	0.249	0.249	0.249	0.248
MSFE:	0.064	0.131	0.171	0.228	0.285	0.291	0.279	0.260	0.267
Resolution:	0.195	0.121	0.080	0.032	0.008	0.008	0.011	0.001	0.003
MS calibration:	0.011	0.006	0.008	0.015	0.047	0.053	0.042	0.010	0.014
Scaled resolution $\in [0,1]$:	0.783	0.485	0.320	0.128	0.032	0.031	0.043	0.004	0.012
Calibration p-value	0.000	0.635	0.847	0.649	0.115	0.024	0.081	0.102	0.000
Resolution p-value	0.000	0.000	0.000	0.009	0.724	0.691	0.473	0.103	0.025
	Inflation: target band								
Horizon:	0	1	2	3	4	5	6	7	8
Outcome Variance:	0.076	0.078	0.079	0.081	0.082	0.084	0.086	0.088	0.090
MSFE:	0.024	0.023	0.058	0.086	0.095	0.093	0.096	0.103	0.116
Resolution:	0.057	0.062	0.042	0.011	0.001	0.001	0.000	0.001	0.002
MS calibration:	0.005	0.008	0.022	0.021	0.016	0.012	0.009	0.013	0.024
Scaled resolution $\in [0,1]$:	0.744	0.792	0.526	0.134	0.018	0.016	0.006	0.010	0.024
Calibration p-value	0.070	0.000	0.164	0.013	0.000	0.000	0.000	0.000	0.000
Resolution p-value	0.000	0.000	0.000	0.238	0.314	0.411	0.601	0.489	0.438

Figure 1
Implied probability forecasts and probability integral transforms

(i) GDP growth vs. threshold: 2010 Q1 data (left); preliminary data (middle); PIT (right)



(ii) Inflation vs. threshold: inflation within bands (left) or below target (middle); PIT (right)

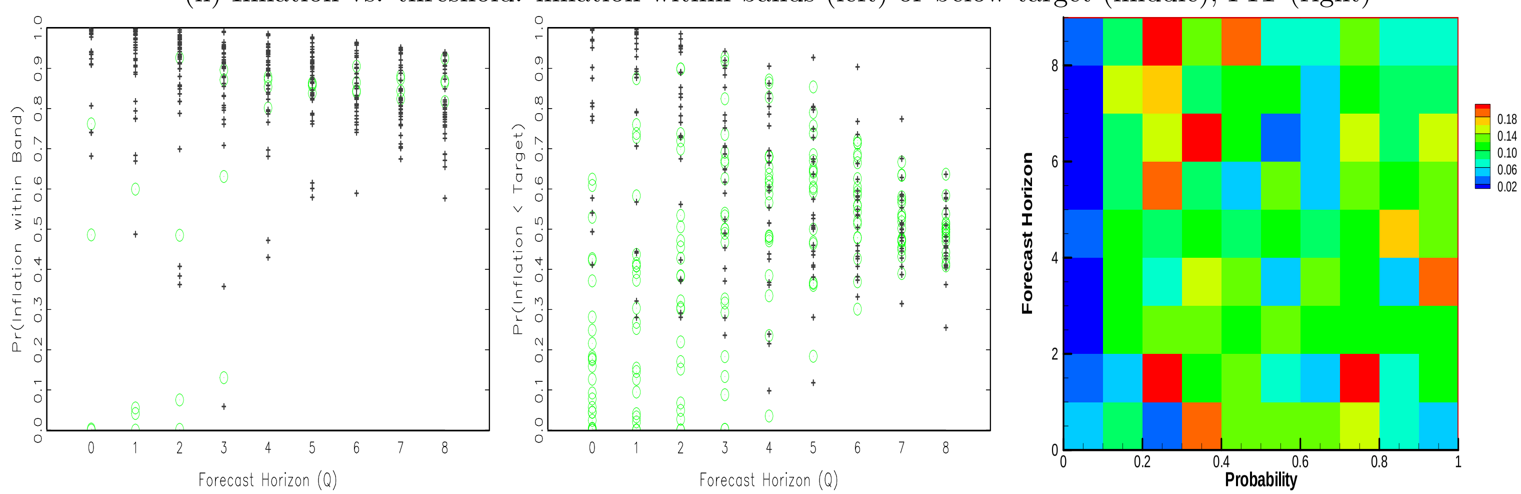
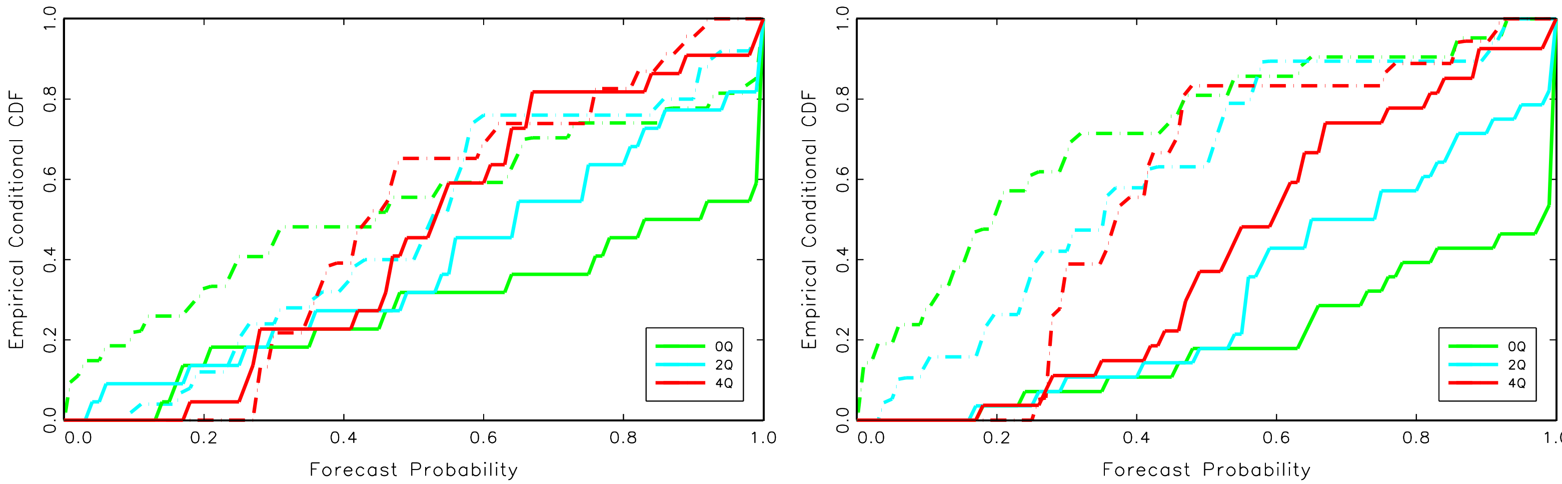


Figure 2

Empirical CDF's of implied probability forecasts from fan charts

(i) GDP growth vs. threshold; market interest rates, 2010 Q1 (left) and preliminary (right) data



(ii) Inflation vs. threshold: market interest rates, cases of inflation within bands (left) and below target (right)

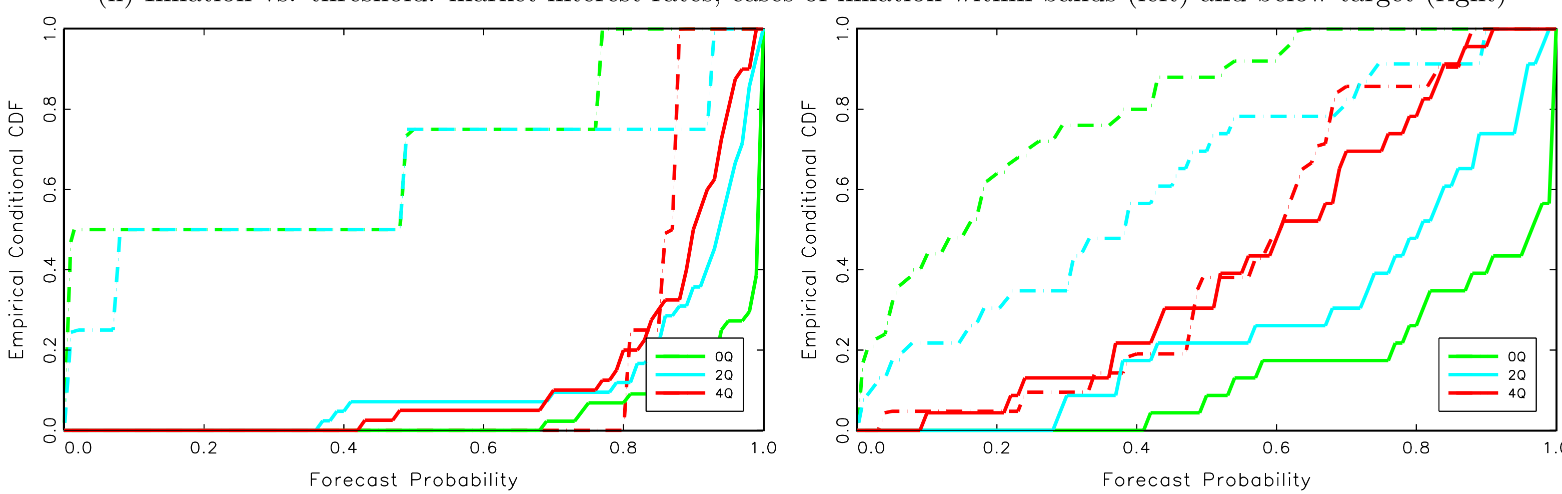
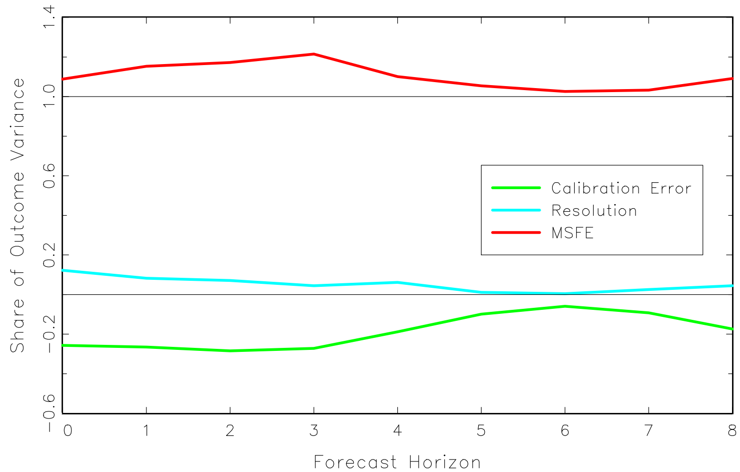
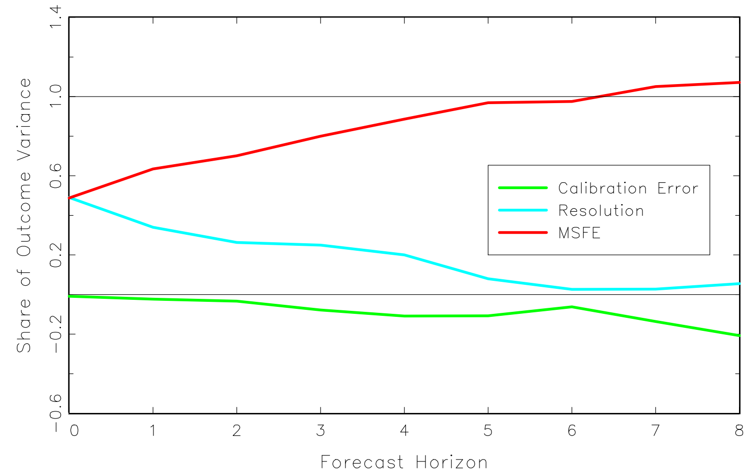


Figure 3

Decomposition of probability forecast MSE

(i) GDP growth vs. threshold; market interest rates, 2010 Q1 (left) and preliminary (right) data





(ii) Inflation vs. threshold: market interest rates, cases of inflation within bands (left) and below target (right)